Attachment 6

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WIND TURBINE STRUCTURAL RESPONSE DUE TO WIND TURBULENCE

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R. W. Thresher
W. E. Holley
Mechanical Engineering Department
Oregon State University
Corvallis, Oregon 97331

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ABSTRACT

This paper presents some approaches to modeling the dynamic response of wind turbine systems to atmospheric turbulence. The first section deals with one possible method for modeling the wind turbulence inputs. The second section looks at the machine response to the turbulence, and shows why the resulting loads should be computed using a coupled dynamic model. The third section examines some of the problems encountered when estimating the fatigue life of a turbine exposed to random atmospheric excitations. In the final section, some suggestions are made for alternate approaches to modeling the effects of turbulence on wind systems.

THE WIND INPUT

It was the goal of the research work at Oregon State University to develop a method for computing the effect of atmospheric turbulence excitations which treated the wind input and the turbine response using the statistical techniques of random vibration theory, and avoid the artificial concept of a discrete deterministic wind gust. A complete statistical description of the turbulent wind field over the rotor disk was computationally impossible, so simplifying assumptions were made. A model was developed that preserved many of the physical pro-

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perties which were known to cause dynamic response of wind turbines, but was computational simple and could be used in either the frequency domain or the time domain. The turbulence field at the rotor disk is approximated with a set of velocity components which are uniform over the rotor disk, and a set of six velocity gradients across the disk. Thus, the model includes all three velocity components, and allows for both horizontal and vertical wind shears in each velocity component. The wind inputs can thus be written as

$$\{V_\infty\} = \begin{Bmatrix} 0 \\ V_w \\ 0 \end{Bmatrix} + \begin{Bmatrix} V_x \\ V_y \\ V_z \end{Bmatrix} + \begin{bmatrix} V_{x,x} & V_{x,y} & V_{x,z} \\ V_{y,x} & V_{y,y} & V_{y,z} \\ V_{z,x} & V_{z,y} & V_{z,z} \end{bmatrix} \begin{Bmatrix} r \sin\Omega t \\ 0 \\ r \cos\Omega t \end{Bmatrix} \tag{1}$$

where the mean wind is in the y direction, the commas imply differentiation with respect to the coordinate of the following subscript, and the uniform velocity terms, V_i, and the linear gradient terms, $V_{i,j}$, are functions of time.

In order to simplify the correlation model, and because certain combinations of the gradient terms in the plane of the rotor always appear together in the linearized aerodynamic relationships, the following quantities are defined.

$$\begin{aligned} \gamma_{zx} &= \frac{1}{2} (V_{z,x} - V_{x,z}), & \bar{\gamma}_{zx} &= \frac{1}{2} (V_{z,x} + V_{x,z}) \\ \varepsilon_{zx} &= \frac{1}{2} (V_{z,z} - V_{x,x}), & \bar{\varepsilon}_{zx} &= \frac{1}{2} (V_{z,z} + V_{x,x}) \end{aligned} \tag{2}$$

Then the turbulent velocity at the rotor can be rewritten as

$$(3)\quad \left\{V_\alpha\right\} = \begin{Bmatrix} V_x + (\bar{\varepsilon}_{zx} - \varepsilon_{zx})\ r\ \sin\Omega t + (\bar{\gamma}_{zx} - \gamma_{zx})\ r\ \cos\Omega t \\ V_w + V_y + V_{y,x}\ r\ \sin\Omega t + V_{y,z}\ r\ \cos\Omega t \\ V_z + (\bar{\gamma}_{zx} + \gamma_{zx})\ r\ \sin\Omega t + (\bar{\varepsilon}_{zx} + \varepsilon_{zx})\ r\ \cos\Omega t \end{Bmatrix}$$

where the nine turbulence inputs: V_x, V_y, V_z, $V_{y,x}$, $V_{y,z}$ and γ_{zx}, ε_{zx}, $\bar{\gamma}_{zx}$, $\bar{\varepsilon}_{zx}$, vary with time and can be shown to be statistically uncorrelated.

A correlation model for the various velocity components is derived using the Von Karman isotropic turbulence model to obtain the correlation between velocities at spatially separated points. Using this correlation model for the turbulence field, the velocity at the rotor disk is approximated by the time dependent uniform and gradient terms of Eq. (2). These terms are chosen to minimize the expected error between the true velocity and the approximate velocity over a region the size of the rotor disk. Furthermore, the power spectral densities are approximated by a simple rational form which corresponds to an exponentially correlated random process, and can be easily used analytically, or for time domain simulations. This model is conveniently expressed by the stochastic differential equation

$$(4)\quad \dot{u} + \hat{a}u = \hat{b}w$$

where

u = instantaneous value of one of the terms $V_x, \ldots, V_{y,x}, \ldots, \gamma_{xz}$, etc.

w = nondimensional white noise with power spectral density $S_w = \sigma^2 L/V_w^3$

$\hat{a}$ = $\frac{V_w}{L} a_*$

$$\hat{b} = \begin{cases} \dfrac{V_w^2}{L}\, b_* & \text{for uniform terms} \\ \\ \dfrac{V_w^2}{LR}\, b_* & \text{for gradient terms} \end{cases}$$

σ^2 = turbulent velocity component variance

L = turbulence integral scale

V_w = mean wind speed

R = rotor disk radius

where the nondimensional coefficients a_* and b_* are tabulated for a wide variety of turbine size to length scale ratios, (R/L). A detail development of this model, as well as, some typical results have been documented in references (1,2).

At this time, work is underway to improve this wind model by adding terms which allow for a quadratic variation in the longitudinal component of the turbulence. This effort is to be completed in September, 1982.

THE TURBINE RESPONSE

The wind turbine model is shown schematically in Figure 1. Both the rotor and the nacelle are assumed to be rigid bodies which move in unison, except for the spinning rotor. Due to tower flexibility, the nacelle and rotor are free to translate in a plane parallel to the ground and rotate about the top of the tower in pitch and yaw. The yaw angle of the rotor axis is defined by the angle, ϕ, and the pitch angle by χ. The lateral translation, U, is in the x direction, while the V translation is in the y direction along the rotor axis. The rotor spin velocity is given by $\Omega + \dot{\Psi}$, where Ω is the mean rotation rate and $\dot{\Psi}$ is some small fluctuation. For the case of a turbine with a three-bladed rigid rotor, the basic principles of Newtonian mechanics and linear, quasi-steady aerodynamics give motion equations of the form

Table 1. Wind Model Assumptions and Important Features

1. The velocity components are correlated using the Von Karman isotropic turbulence model.
2. The turbulent velocity field at the rotor disk is approximated using three uniform terms plus six gradient terms.
3. Each of these nine turbulence inputs is modeled as a stationary, exponentially correlated random process, which can be represented by a first order linear differential equation.
4. Velocity shears caused by turbulence seem to result in significant turbine response and are modeled for all the velocity components.
5. Three wind parameters are required to model a specific site: the mean wind, the turbulent velocity component variance, and the turbulence length scale.
6. The model can be used to perform analysis in the frequency domain, as was done for the results which will be presented here, or the differential equations of the wind model can be used to drive any type of time domain simulation.

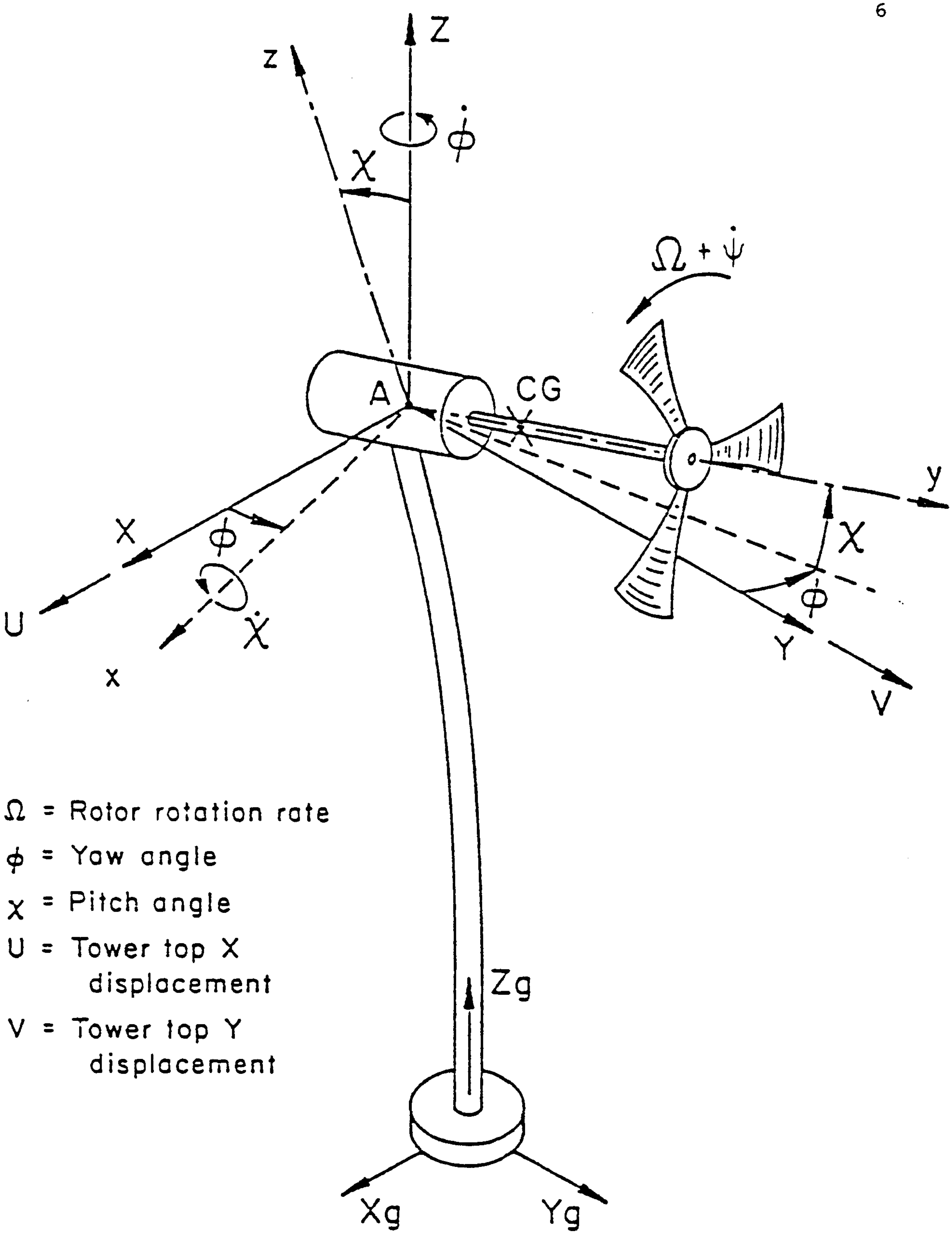


Figure 1. The Turbine Model

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{Q_1\} + [F]\{u\} \tag{5}$$

where [M], [C], [K], and [F] are the inertia, damping, stiffness, input coefficient matrices,

$$\{X\}^T = (U,V,\phi,\chi,\Psi) = \text{displacement coordinate}$$
$$\{Q_1\}^T = (0,T,0,Q,0) = \text{steady state}$$
$$\{u\}^T = (V_x,V_y,V_z,V_{y,x},V_{y,z},\gamma_{xz},\varepsilon_r,\bar{\gamma}_r,\bar{\varepsilon}_{xz}) = \text{wind inputs}$$
$$\varepsilon_r = \varepsilon_{zx}\cos 3\Omega t + \bar{\gamma}_{zx}\sin 3\Omega t$$
$$\bar{\gamma}_r = -\varepsilon_{zx}\sin 3\Omega t + \bar{\gamma}_{zx}\cos 3\Omega t$$

The terms ε_r and $\bar{\gamma}_r$ come from the three-bladed sums of the aerodynamic forces that involve $\sin(2\Omega t)$ and $\cos(2\Omega t)$.

Discarding the steady terms, it is convenient to transform these turbine equations to the state space form, and to augment them with the nine wind input equations. This forms a single set of equations with white noise as the driving input. These may be written as

$$\{\dot{x}\} = [A]\{x\} + [B]\{w\} \tag{6}$$

$$\{y\} = [C]\{x\}$$

where

$$\{x\} = \begin{Bmatrix} \{X\} \\ \{\dot{X}\} \\ \{u\} \end{Bmatrix} \qquad [A] = \begin{bmatrix} [0] & [I] & [0] \\ -[M]^{-1}[K] & -[M]^{-1}[K] & [M]^{-1}[F] \\ [0] & [0] & [\hat{a}] \end{bmatrix}$$

$$[B] = \begin{bmatrix} [0] \\ [\hat{b}] \end{bmatrix} \qquad \{y\} = \begin{Bmatrix} F_x \\ F_y \\ M_z \\ M_x \\ \text{Power} \\ \{x\} \end{Bmatrix} = \text{outputs}$$

$[C] \;\; = \;\;$ response matrix

The displacements represented by $\{x\}$ and velocities given by $\{\dot{x}\}$ are deviations from the steady values. The outputs $\{y\}$ are selected by the user, and depend on the coefficients of the response matrix $[C]$. With this formulation it is a relatively straightforward numerical procedure, to determine the complex eigenvalues of the A matrix and then to compute the modal matrix, which is made up of the associated eigenvectors. The modal matrix can then be used to decouple the equations of motion so that transfer functions between any of the nine white noise inputs and any output, y_i, may be easily computed. These transfer functions account for differences in the energy level for the turbulence inputs, $\{u\}$, so that a comparison of the transfer function magnitudes provides a direct estimate of relative importance. The final result uses the central equation from random vibration theory Eq. (7), which states that the spectral density for any of the outputs $\{y\}$ will be given by

$$\{S_y(\omega)\} = [|H_{yw}(\omega)|^2] \;\; \{S_w\} \tag{7}$$

for uncorrelated inputs. In this equation, $\{S_y(\omega)\}$ is the spectral density of the outputs $\{y\}$, $[|H_{yw}(\omega)|^2]$ is the matrix consisting of elements which are the

Table 2. Mod-G Characteristics

Rotor Characteristics:		
Rotor Radius		150 ft
Blade Chord (linear taper)		7.74 ft at hub to 3.15 ft at tip
Coning Angle		4°
Blade Twist (linear)		8°
System Frequencies:		
1st Bending (fore-aft)	(1.5Ω)	2.7 rad/s
2nd Bending (fore-aft)	(7.5Ω)	13.7 rad/s
1st Bending (side-to-side)	(1.6Ω)	2.9 rad/s
1st Torsion	(4.9Ω)	9.0 rad/s
Aerodynamic Properties:		
Lift Curve Slope		5.73
Drag Coefficient, C_{D_0}		.008
Stall not Modeled		
Operating Conditions:		
Wind Velocity	(1.833 rad/s)	20 MPH
Rotor Speed		17.5 RPM
Pitch Setting at Tip		-6.2°
Turbulence Length Scale		500 ft
Rms turbulent intensity		2.44 ft/s
Approximate Power Output		1.1 MW

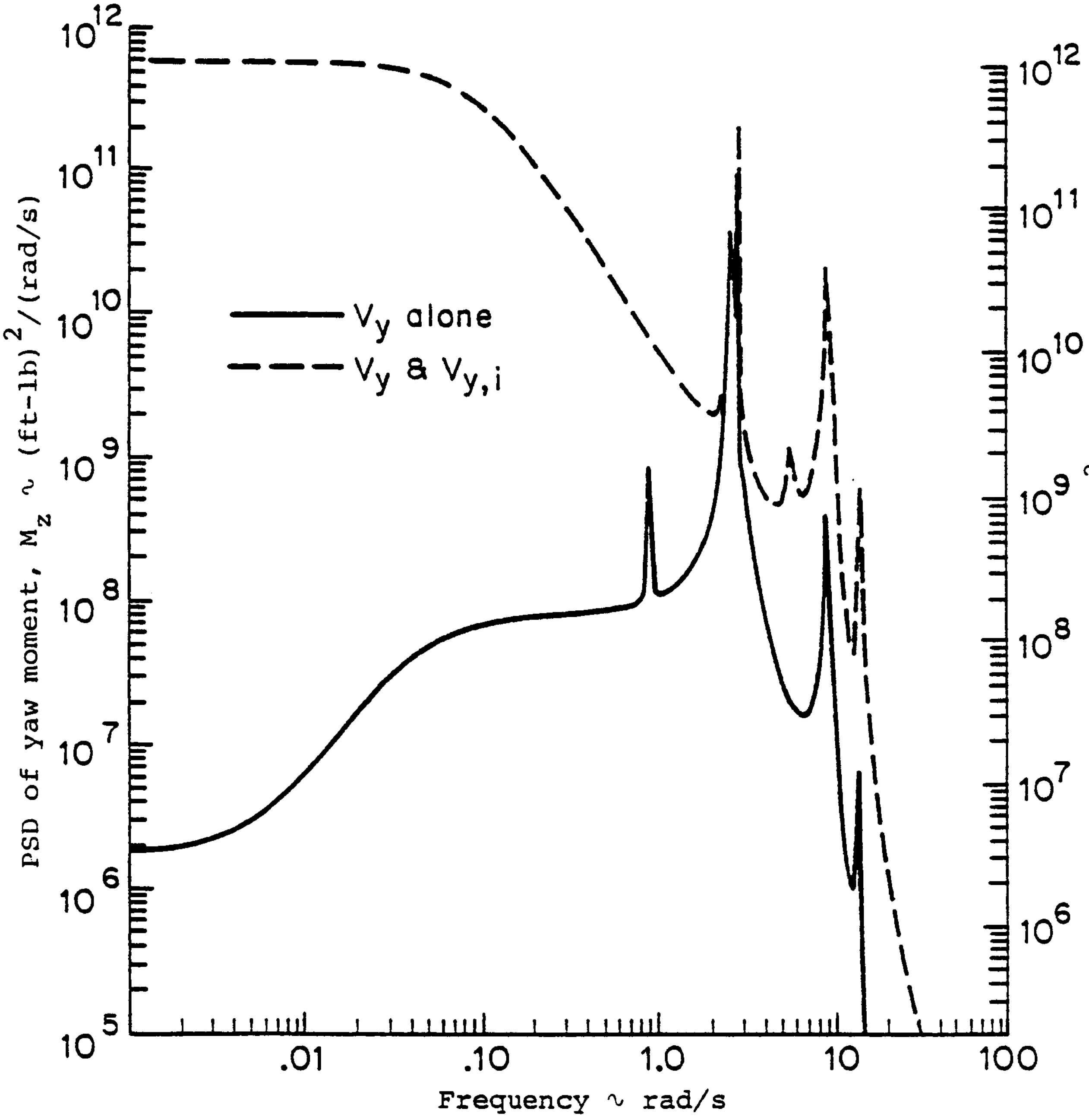


Figure 2. The effect of the gradients $v_{y,x}$ and $v_{y,z}$ on yaw moments for Mod-G using the equilibrium wake.

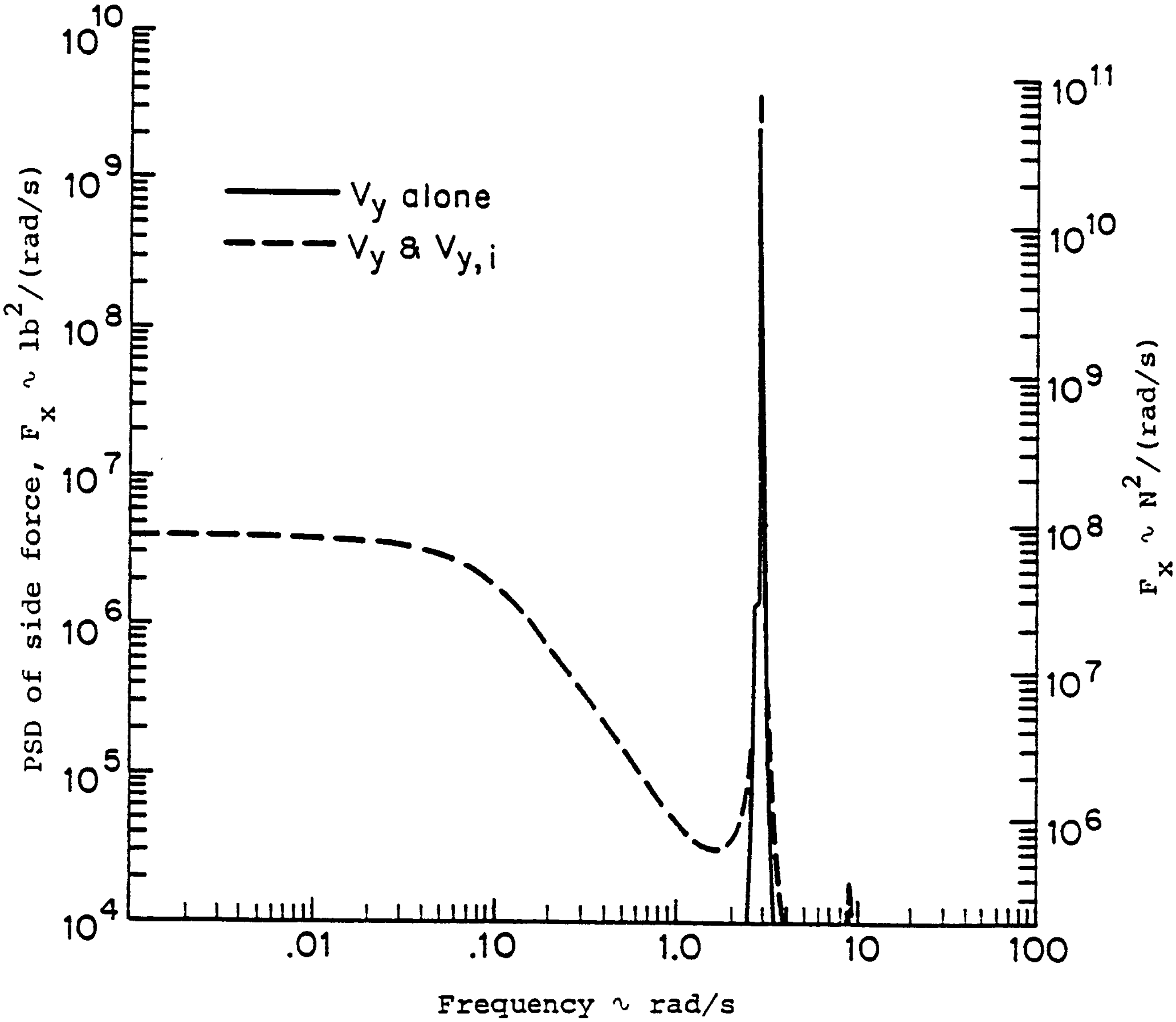


Figure 3. The effect of the gradients $v_{y,x}$ and $v_{y,z}$ on the side force for Mod-G using the equilibrium wake.

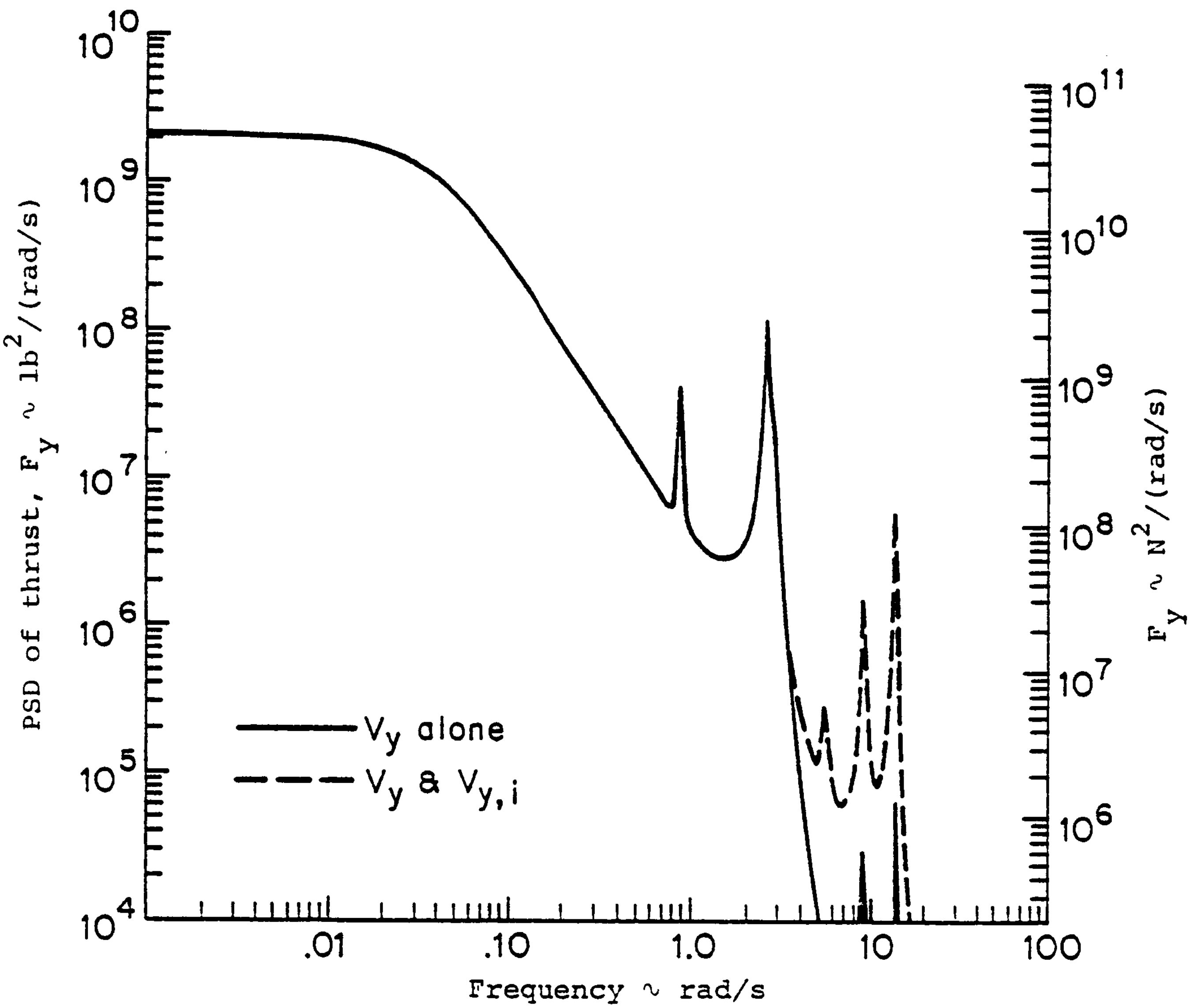


Figure 4. The effect of the gradients $v_{y,x}$ and $v_{y,z}$ on thrust for the Mod-G using the equilibrium wake.

square of the transfer function magnitude and $\{S_w\}$ is the flat spectral density of the white noise driving inputs, which are all equal.

Using this procedure a large wind turbine called the Mod-G was analyzed. The Mod-G is a 2.5 MW turbine with a three-bladed rotor located upwind of the tower, and is designed for fixed-yaw operation. The specific characteristics of this system are shown in Table 2. Figures 2 through 4 present the computation results for the Mod-G turbine.

The primary objective of this work was to identify the features of turbulence which are most important in wind turbine design. In an effort to focus on these key features, the response at specific system frequencies was broken down into fractional contributions from each turbulence input. The most significant results of these calculations are tabulated in Table 3.

From these results it seems clear that the most important inputs are the longitudinal turbulence component, V_y, the two associated gradient terms $V_{y,x}$ and $V_{y,z}$.

To examine this conclusion more closely, consider again Figures 2 through 4, which present plots of power spectral densities for the various response variables using, first, only the turbulence input V_y, and then comparing it with the results when the two gradients $V_{y,x}$ and $V_{y,z}$ are added to the input. The figures clearly show that the response is significantly underestimated unless the turbulence gradient terms $V_{y,x}$ and $V_{y,z}$ are included.

There are two simple conclusions which arise from the results presented here. First, the turbine response to atmospheric turbulence should be obtained using a coupled dynamic model which inputs the wind excitations over the appropriate frequency range. If this is not done, then all of the turbine natural frequencies will not be excited in a realistic manner. In addition, it is essential to model the spatial variations in wind velocity caused by turbulence.

Table 3. Fractional Response Contributions of the Turbulence Inputs for the Mod-G Using the Equilibrium Wake

Response/Input	V_y	$V_{y,x}$	$V_{y,z}$	ε_r	$\bar{\gamma}_r$	Other
Frequency ≈ 0						
Side Force, F_x	.0	.06	.92	.0	.0	.02
Thrust, F_y	1.0	.0	.0	.0	.0	.0
Yaw Moment, M_z	.0	.97	.0	.0	.0	.03
Pitch Moment, M_x	.0	.0	.97	.0	.0	.03
Frequency = 2.69 (Fore-Aft Bending)						
Side Force, F_x	.93	.04	.02	.0	.0	.01
Thrust, F_y	.78	.0	.21	.0	.0	.01
Yaw Moment, M_z	.77	.02	.20	.0	.0	.01
Pitch Moment, M_x	.78	.0	.21	.0	.0	.01
Frequency = 3Ω = 5.5						
Side Force, F_x	.02	.39	.09	.25	.24	.01
Thrust, F_y	.01	.02	.19	.38	.39	.01
Yaw Moment, M_z	.02	.31	.0	.34	.32	.01
Pitch Moment, M_x	.01	.02	.20	.38	.39	.0
Frequency = .89 (Drive Train)						
Side Force, F_x	.0	.08	.90	.0	.0	.02
Thrust, F_y	1.0	.0	.0	.0	.0	.0
Yaw Moment, M_z	.11	.87	.01	.0	.0	.01
Pitch Moment, M_x	.01	.0	.97	.0	.0	.02

FATIGUE DAMAGE

For a structure exposed to a Gaussian narrow band loading, there are several approaches to computing the expected life. The most straightforward is the Palmgren-Miner rule. This rule states that, if n_i cycles occur at stress level s_i, and at constant amplitude, it would take N_i cycles at this level for failure, then the fractional damage at s_i is (n_i/N_i). Failure is expected when the sum of all the fractional damages equals unity. That is when

$$\sum_i = (n_i/N_i) = 1$$

From this it is possible to determine the time to failure as

$$T = \frac{1}{\nu_o^+ \int_o^\infty \frac{P_p(s)\ ds}{N(s)}}$$

where $N(s)$ = the number of cycles to failure at s.

ν_o^+ = $\frac{1}{2\pi} \frac{\sigma_{\dot{s}}}{\sigma_s}$ = zero crossing frequency.

$P_p(s)$ = $\frac{s}{\sigma_s} e^{-s^2/2\sigma_s^2}$ = probability density function of stress peaks.

σ_s^2 = the variance of the stress, s.

$\sigma_{\dot{s}}^2$ = the variance of $\dot{s}$.

The variance can be computed from the power-spectral-density by direct integration or directly from the system governing equations. For a single response variable, this would be

$$\sigma_s^2 = \int_{-\infty}^{\infty} S_s(\omega)\, d\omega$$

and

$$\sigma_{\dot{s}}^2 = \int_{-\infty}^{\infty} \omega^2 S_s(\omega)\, d\omega$$

where $S_s(\omega)$ is the power-spectral-density for the stress s. Note that these two integrals represent areas under spectral density curves.

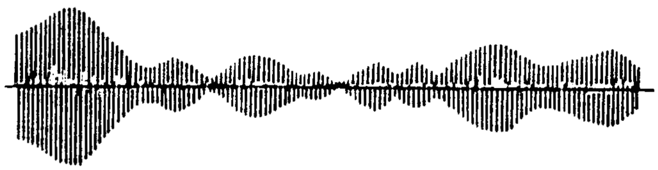
Alternately, fracture mechanics techniques can be used. Fatigue-crack-growth data can be conveniently represented by the equation

$$\frac{da}{dN} = C(\Delta K_I)^m$$

where a is crack length; N is the number of cycles; C is a material constant; ΔK_I is the range of the stress intensity factor; and m is an exponent in the range 2-4.

Using this approach Pook and Greenan (3) have performed statistical computations similar to those presented above for Miner's rule, and compared the results with a limited amount of experimental data for mild steel under narrow-band random loading. Results for this work are reproduced in Figure 7. The spread in the predictive results would seem to indicate that there is still some research work that needs to be done.

These results apply only for narrow-band random loading. For wind systems the response is generally wide-band with major contributions at the natural frequencies of the system. While the above methods can be modified to the more complex situation of wide-band loading as Holley (4) has demonstrated, the linear damage rules do not predict damage nearly so well as for narrow-band loading. In general, prediction fatigue life under wide-band random loads is considered to be a research topic of significant difficulty.



Narrow-band random loading.

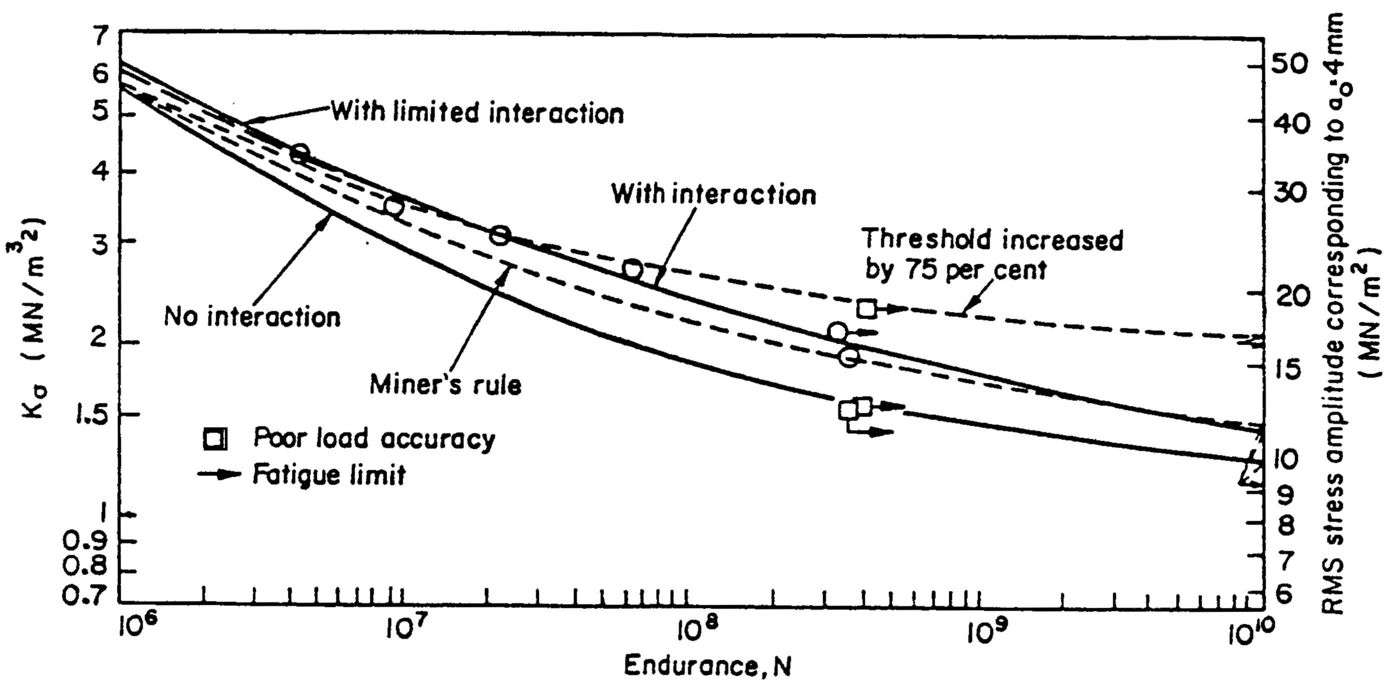


Figure 5. Results of comparison and loading from reference (3).

CONCLUSIONS AND RECOMMENDATIONS

This paper has put forth the following major points:

1. For computation of wind turbine loads, the turbulence inputs must include terms which generate a nonuniform spatial distribution of velocity over the rotor disk, otherwise important excitations will be lost.
2. The procedures used to compute the dynamic loads caused by turbulence must allow a full dynamic response to these inputs. Quasi-steady computation and the use of discrete deterministic gusts will probably give misleading results.
3. The response of wind turbines to turbulence inputs tends to be wide-band, and the usual fatigue damage rules may not provide accurate estimates of structural life for wide-band loading. This problem is not, however, unique to wind systems. It is a generic problem common to many mechanical systems, and should be classified as a "basic research issue" of significant importance to the success of wind energy systems.

In addition to these major points, the authors would like to make the following recommendations:

1. At this point in time, there is little experimental data, in the form of spectral-density plots of machine loads, to use as a guide for model development and setting design criteria. This type of data would be very helpful, and should be developed and published. For a complete picture, the associated wind data is also necessary.
2. Because the governing equation for two-bladed wind turbines contains periodic coefficients, it would appear at first that frequency domain techniques are not practical. However, Holley and Bahrami (5) have extended the analysis

presented here using Floquet theory to periodic linear systems. In addition, under some simplifying assumptions, it may be possible to use the nonlinear time domain computer codes already developed for dynamic analysis of turbines to compute a set of transfer functions relating a specific wind turbulence input and any desired responses. For example, a nonlinear code could be used to compute the response to a mean wind and a suddenly applied linear gradient across the disk, $V_{y,x}$, where the gradient time history is a square wave of several cycles with each cycle of shorter duration to fully excite higher turbine natural frequencies. From the time history of the input and any particular output, a transfer function could then be computed numerically. This would provide a set of linearized transfer functions which would contain the proper frequencies, but would in some sense average-out the effect of the periodic coefficients. This approach has the advantage of using the existing codes and allowing the turbulence calculations to be done separately, but would need to be fully validated for design use.

3. Some form of time domain analysis should also be developed. However, it seems likely that the computation time for a full system simulation will be long. For this reason, it may be best to model only the power train and rotor system to validate the turbulence input modeling with field test data. After validation of the technique, a more comprehensive turbine system model could be developed.

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